GRACEFUL LABELING AND ρˆ LABELING ON THE 8-BINTANG GRAPH

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GRACEFUL LABELING AND $\hat{\rho}$ LABELING ON THE *8-BINTANG* GRAPH

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Abstract. *A graph $G = (V, E)$ is ordered set V and E, where V is finite, non-empty set of objects called vertices or nodes, and E is the set of arcs. Graceful labeling is an injective function f of the set of vertices V to the set of numbers $\{0, 1, 2, \ldots, |E|\}$ which induces the bijective function f' from the set of arc E to the set of number $\{1, 2, \ldots, |\mathrm{E}|\}$, where each arc $uv \in E$ with node $u, v \in V$ apply $f'(uv) = |f(u) - f(v)|$, is aligned with the variations and modifications of graceful labeling. The basic idea of constructing graceful labeling and $\hat{\rho}$ labeling on an* 8-Bintang *graph begins with* A-Bintang *and* H-Bintang*. which is then referred to as a star alphabet graph with the question how if the number is given a star graph S_n then a graph constructed from 2 circle graph where one vertex of the circle graph becomes the center of the graph while the other node is given graph Star S_n. In this paper is given for* 8-Bintang *graph with C_4 and C_3 for n even.*

1. INTRODUCTION

A graph G is an ordered pair (V, E) with V being the set of nodes or vertices and E is a set of multisets consisting of two elements namely element V and element E each called nodes and arcs. Labeling on a graph is essentially giving a certain value to a node and or an arc that satisfies certain rules as well. in [4] noted for the last 50 years until data dated November 13, 2010 there are 1198 articles that discuss about the various kinds of labeling[3]. In general, Graceful labeling on graph G (V,E)is an injective

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function f of the set of vertices V to the set of numbers $\{0, 1, 2, \ldots, |E|\}$, which induces the bijective function f from the set of arc E to set of numbers $\{1, 2, \ldots, |E|\}$ Where each arc $uv \in E$ with the vertex $uv \in E$ apply $f'(uv) = |f(u) - f(v)|$[1,2,5,7].

Labeling $\hat{\rho}$ is a modification of the graceful labeling of the injective function g of the set of vertices V to the set of numbers $\{0, 1, 2, \ldots, |E|+1\}$ that induce the bijective function g from the set of arc E to set of numbers $\{1, 2, \ldots, |E|\}$, where each arc $uv \in E$ with node $u, v \in V$ apply $g(uv) = |g(u)g(v)|$[1,2,3,5,7].

The cycle graph of length n is a graph with n vertices $\{v_1, v_2, \ldots, v_n\}$ and $\{v_1v_2, v_2v_3, \ldots, v_{(n-1)}v_n, v_nv_{(n+1)}\}$[6]. In Figure 1.1 we have examples of circular graphs C_3, C_4, C_5.

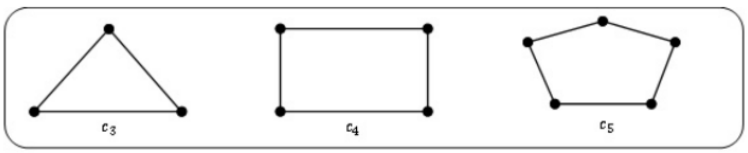

Figure 1: Circular Graph (Cycle)

The S_n star graph is a graph constructed from a central node and then adds a number of leaf nodes to the central node. The star graph has (n+1) node and arc n.

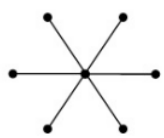

Figure 2: Star Graph

The star graph is a subclass of a tree graph, since the star graph has no circular subgraph.

The *8-Bintang* graph with the builder C_3 (Fig. 3) is a graph constructed from two circular graphs C_3 where one of the vertices of the circle graph C_3 becomes the center of the graph while the other node is given the star S_n.

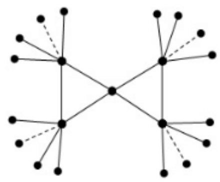

Figure 3: $8 - Bintang$ with C_3

The *8-Bintang* graph with the builder C_4 (Fig. 4) is a graph constructed from two circular graphs where one of the vertices of the circle graph becomes the center of the graph while the other node is given the star S_n.

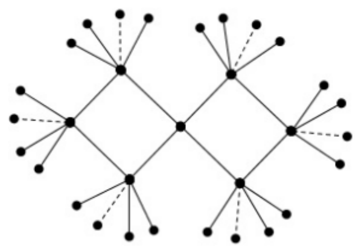

Figure 4: $8 - Bintang$ with C_4

So that can be given the definition of *8-Bintang* graph is a graph built from 2 pieces of circle graph where one of the vertices of the circle graph becomes the center of the graph while the other node is given the star graph S_n.

2. GRACEFUL LABELING ONn *8-BINTANG* GRAPH

In this section will be given graceful labeling construction on 8-Star graph.

Theorem 2.1 *The* 8-Bintang *Graph with* C_3 *has a graceful labeling for* n *even*

Proof 2.1 *Suppose the notation of the* 8-Bintang*graph node with* C_3 *is given in Figure 5*

In Figure.5 above shows that the set of 8-Bintang *graph nodes is* $\{c_0, c_1, \ldots, c_4, v_1^1, \ldots, v_n^1, v_1^2, \ldots, v_n^2, v_1^3, \ldots, v_n^3, v_1^4, \ldots, v_n^4\}$ *and the arc set of* 8-Bintang *graph*

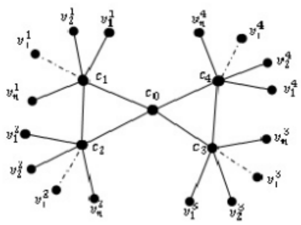

Figure 5: Notasi 8 − *Bintang* Graph with C_3

is $\{c_0c_1, c_1c_2, c_2c_0, c_0c_3, c_3c_4, c_4c_0, c_1v_1^1, c_1v_2^1, \ldots, c_1v_n^1, c_2v_1^2, \ldots, c_2v_n^2, c_3v_1^3, \ldots, c_3v_n^3, c_4v_1^4, \ldots, c_4v_n^4\}$ *so that the number of elements* V *and* E *are* $|V| = 4n + 5$ *and* $|E| = 4n + 6$ *respectively*

Defined labeling by using the f *notation for the node as follows:*

$$f(c_0) = n + 1 \tag{1}$$

$$f(c_1) = 0 \tag{2}$$

$$f(c_2) = 3n + 6 \tag{3}$$

$$f(c_3) = n + 2 \tag{4}$$

$$f(c_4) = 3n + 5 \tag{5}$$

$$f(v_i^1) = 4n + 6 - i; \qquad i = 1, 2, \ldots, n \tag{6}$$

$$f(v_i^2) = i; \qquad i = 1, 2, \ldots, n \tag{7}$$

$$f(v_i^3) = \begin{cases} n + 2 + 2i; & i = 1, 2, \ldots \frac{1}{2}n \\ n + 3 + 2i; & i = \frac{1}{2}n + 1, \frac{1}{2}n + 2, \ldots, n \end{cases} \tag{8}$$

$$f(v_i^4) = \begin{cases} n + 1 + 2i; & i = 1, 2, \ldots \frac{1}{2}n + 1 \\ n + 2 + 2i; & i = \frac{1}{2}n + 2, \frac{1}{2}n + 3, \ldots, n \end{cases} \tag{9}$$

The labeling f *defined in equation (1) - (9), labeling each node member of an* 8-Bintang *graph is an injective mapping from* V *to set* $\{0, 1, \ldots, |E|\}$. *Each* $uv \in E$ *arc is labeled by the labeling of* f', *induced by the labeling of vertices through the equation* $f'(uv) = |f(u) - f(vv)|$ *On an* 8-Bintang *graph expressed as follows:*

$$f'(c_0c_1) = |f(c_0) - f(c_1)| = |(n+1) - (0)| = 4 + 1 \quad (10)$$
$$f'(c_1c_2) = |f(c_1) - f(c_2)| = |(0) - (3n+6)| = 3n + 6 \quad (11)$$
$$f'(c_2c_0) = |f(c_2) - f(c_0)| = |(3n+6) - (n+1)| = 2n + 5 \quad (12)$$
$$f'(c_0c_3) = |f(c_0) - f(c_3)| = |(n+1) - (n+2)| = 1 \quad (13)$$
$$f'(c_3c_4) = |f(c_3) - f(c_4)| = |(n+2) - (3n+5)| = 2n + 3 \quad (14)$$
$$f'(c_4c_0) = |f(c_4) - f(c_0)| = |(3n+5) - (n+1)| = 2n + 4 \quad (15)$$
$$f'(c_1v_i^1) = |f(c_1) - f(v_i^1)| = |(0) - (4n+6-i)| = 4n+6-i; \qquad i = 1, 2, \ldots, n \quad (16)$$
$$f'(c_2v_i^2) = |f(c_2) - f(v_i^2)| = |(3n+6) - (i)| = 3n + 6 - i; \qquad i = 1, 2, \ldots, n \quad (17)$$

Since $f(v_i^3)$ *has two conditions such as equation (8), the arc value adjacent to the node* $f'(v_i^3)$ *can be expressed according to the conditions specified in (8) for* $i = 1, 2, \ldots, 1/2n$ *is obtained by the equation (18) and for the condition kondisi* $i = 1/2n + 1, 1/2n + 2, , n$ *we obtain the equation (19) below*

$$f'(c_3v_i^3) = |f(c_3) - f(v_i^3)| = |(n+2) - (n+2+2i)| = 2i; \qquad i = 1, 2, \ldots, \frac{1}{2}n \quad (18)$$
$$f'(c_3v_i^3) = |f(c_3) - f(v_i^3)| = |(n+2) - (n+3+2i)| = 2i+1 \quad i = \frac{1}{2}n+1, \frac{1}{2}n+2, \ldots, n \quad (19)$$

Similarly to obtain $f'(c_4v_i^4)$ *with the conditions expressed in equation (9). The arc* $f'(c_4v_i^4)$ *as in the following (20 - 21) equations*

$$f'(c_4v_i^4) = |f(c_4) - f(v_i^4)| = |(3n+5) - (n+1+2i)| = 2i \quad i = 1, 2, \ldots, \frac{1}{2}n+1 \quad (20)$$
$$f'(c_4v_i^4) = |f(c_4) - f(v_i^4)| = |(3n+5) - (n+2+2i)| = 2i+1 \quad i = \frac{1}{2}n+2, \frac{1}{2}n+, \ldots, n \quad (21)$$

Based on f *labeling defined in equation (1) - (9) each vertex has a different label and is a subset of numbers* $\{0, 1, 2, \ldots, |E|\}$ *and always does not contain the node labeling* $2n + 4$ *and* $3n + 4$*. Then the labeling of* f' *induced by the vertex labeling* f*, gives different values to each arc as in (10) - (21) which is the set of numbers* $\{1, 2, \ldots, |E|\}$*. Based on the above result, then* f *is the graceful labeling on* 8-Bintang *graph with* n *even.*

Theorem 2.2 *The* 8-Bintang *Graph with C_4 has a graceful labeling*

Proof 2.2 *Suppose the notation of the* 8-Bintang*graph node with C_3 is given in Figure.6*

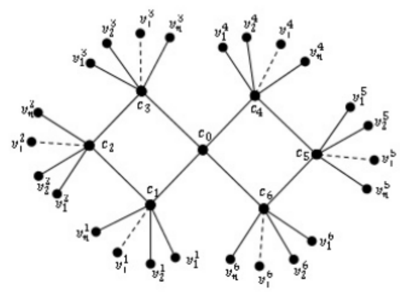

Figure 6: Notasi $8 - \mathit{Bintang}$ Graph with C_4

In Figure.6 above shows that the set of 8-Bintang *graph nodes is* $\{c_0, c_1, \ldots, c_6, v_1^1, \ldots, v_n^1, v_1^2, \ldots, v_n^2, v_1^3, \ldots, v_n^3, v_1^4, \ldots, v_n^5, v_1^5, \ldots, v_n^5, v_1^5, \ldots, v_n^6\}$ *and the arc set of* 8-Bintang *graph is* $\{c_0c_1, c_1c_2, c_2c_3, c_3c_0, c_0c_4, c_4c_5, c_5c_6, c_6c_0, c_1v_1^1, c_1v_2^1, \ldots, c_1v_n^1, c_2v_1^2, \ldots, c_2v_n^2, c_3v_1^3, \ldots, c_3v_n^3, c_4v_1^4, \ldots, c_4v_n^4, c_5v_1^5, \ldots, c_5v_n^5, c_6v_1^6, \ldots, c_6v_n^6\}$ *so that the number of elements V and E are $|V| = 4n + 5$ and $|E| = 4n + 6$ respectively*

Defined labeling by using the f notation for the node as follows:

$$f(c_0) = n + 1 \tag{22}$$

$$f(c_1) = 5n + 8 \tag{23}$$

$$f(c_2) = 0 \tag{24}$$

$$f(c_3) = 2n + 4 \tag{25}$$

$$f(c_4) = 2n + 3 \tag{26}$$

$$f(c_5) = 2n + 2 \tag{27}$$

$$f(c_6) = 5n + 7 \tag{28}$$

$$f(v_i^1) = i; \qquad i = 1, 2, \ldots, n \tag{29}$$

$$f(v_i^2) = 5n + 8 + i; \qquad i = 1, 2, \ldots, n \tag{30}$$

$$f(v_i^3) = 2n + 5 + i; \qquad i = 1, 2, \ldots, n \tag{31}$$
$$f(v_i^4) = 3n + 6 + i; \qquad i = 1, 2, \ldots, n \tag{32}$$
$$f(v_i^5) = 4n + 6 + i; \qquad i = 1, 2, \ldots, n \tag{33}$$
$$f(v_i^6) = n + 1 + i; \qquad i = 1, 2, \ldots, n \tag{34}$$

The labeling f defined in equation (22) - (34), will label each member node of an 8-Bintang *graph with a builder element c_4 is an injective mapping from V to set $\{0, 1, \ldots, |E|\}$. Each $uv \in E$ arc is labeled by the labeling of f', induced by the labeling of vertices through the equation $f'(uv) = |f(u) - f(vv)|$ On the* 8-Bintang *graph expressed as follows:*

$$f'(c_0c_1) = |f(c_0) - f(c_1)| = |(n + 1) - (5n + 8)| = 4n + 7 \tag{35}$$
$$f'(c_1c_2) = |f(c_1) - f(c_2)| = |(5n + 8) - (0)| = 5n + 8 \tag{36}$$
$$f'(c_2c_3) = |f(c_2) - f(c_3)| = |(0) - (2n + 4)| = 2n + 4 \tag{37}$$
$$f'(c_3c_0) = |f(c_3) - f(c_0)| = |(2n + 4) - (n + 1)| = n + 3 \tag{38}$$
$$f'(c_0c_4) = |f(c_0) - f(c_4)| = |(n + 1) - (2n + 3)| = n + 2 \tag{39}$$
$$f'(c_4c_5) = |f(c_4) - f(c_5)| = |(2n + 3) - (2n + 2)| = 1 \tag{40}$$
$$f'(c_5c_6) = |f(c_5) - f(c_6)| = |(2n + 2) - (5n + 7)| = 3n + 5 \tag{41}$$
$$f'(c_6c_0) = |f(c_6) - f(c_0)| = |(5n + 7) - (n + 1)| = 4n + 6 \tag{42}$$
$$f'(c_1v_i^1) = |f(c_1) - f(v_i^1)| = |(5n+8) - (i)| = 5n+8-i; \qquad i = 1, 2, \ldots, n \tag{43}$$
$$f'(c_2v_i^2) = |f(c_2) - f(v_i^2)| = |(0) - (5n+8+i)| = 5n+8+i; \qquad i = 1, 2, \ldots, n \tag{44}$$
$$f'(c_3v_i^3) = |f(c_3) - f(v_i^3)| = |(2n+4) - (2n+5+i)| = 1+i \qquad i = 1, 2, \ldots, n \tag{45}$$
$$f'(c_4v_i^4) = |f(c_4) - f(v_i^4)| = |(2n+3) - (3n+6+i)| = n+3+i \qquad i = 1, 2, \ldots, n \tag{46}$$
$$f'(c_5v_i^5) = |f(c_5) - f(v_i^5)| = |(2n+2) - (4n+6+i)| = 2n+4+i; \qquad i = 1, 2, \ldots, n \tag{47}$$
$$f'(c_6v_i^6) = |f(c_6) - f(v_i^6)| = |(5n+7) - (n+1+i)| = 4n+6-i; \qquad i = 1, 2, \ldots, n \tag{48}$$

Based on labeling f defined in equation (22) - (34) each vertex has a different label and is a subset of numbers $\{0, 1, 2, \ldots, |E|\}$ and always does not contain the node $2n + 5$ and $3n + 6$. Then labeling f' induced by the *labeling of node f gives different values to each arc as in (35) - (48) which is the set of numbers $\{1, 2, \ldots, |E|\}$. Based on the above results, then f is a graceful labeling for* 8-Bintang *graph.*

3. $\hat{\rho}$ LABELING *8-BINTANG* GRAPH WITH c_3 FOR n EVEN

Corollary 3.1 *The* 8-Bintang *graph with c_3 consists labeling $\hat{\rho}$for n even*

Proof 3.3 *Suppose the* 8-Bintang *graph notation with C_3 is shown in Figure. 5. Using the same way of graceful proofing of Theorem 2.1 by defining the node $g = f$ labeling as equation (1)–(9) and arc labeling $g(uv) = |g(u) - g(v)|$, where $uv \in E$ with $u, v \in V$ we obtain the node labeling from the* 8-Bintang *graph to the subset of numbers $\{0, 1, 2, \ldots, |E| + 1\}$, and arc labeling from the* 8-Bintang *graph to the set of numbers $\{1, 2, \ldots, |E|\}$. So the* 8-Bintang *graph with C_3 has labeling*

Corollary 3.2 *the* 8-Bintang *graph with c_4 is $\hat{\rho}$ labeling*

Proof 3.4 *Suppose the* 8-Bintang *graph notation with C_4 is shown in Figure. 6. Using the same method of proof on Corollary 3.1 that will form $\hat{\rho}$ labeling, which is constructed as equation (22)-(48).*

4. CONCLUSION

In this paper we have given Graceful labeling and $\hat{\rho}$ labeling on *8-Bintang* graph constructed from 2 circular graph where one of the vertices of the circle graph becomes the center of the graph while the other vertices is given S_n star graph. More generally it can be proved that *8-Bintang* graphs have graceful labeling and $\hat{\rho}$ labeling.

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